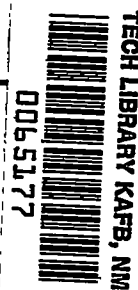


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# NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 2195

A FLIGHT INVESTIGATION AND ANALYSIS OF THE LATERAL-  
OSCILLATION CHARACTERISTICS OF AN AIRPLANE

By Carl J. Stough and William M. Kauffman

Ames Aeronautical Laboratory  
Moffett Field, Calif.



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OSCILLATION CHARACTERISTICS OF AN AIRPLANE

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## SUMMARY

Flight tests were conducted and an analysis made to determine the causes of undesirable dynamic lateral-stability characteristics of an airplane. Various rudder modifications were flight tested with the rudder free and fixed over an indicated airspeed range from approximately 200 to 450 miles per hour. Rudder-hinge-moment and other pertinent data were obtained by flight and wind-tunnel tests.

The oscillation period and time required to damp to one-half amplitude measured in flight are compared with calculated curves in which these characteristics are given as functions of the important rudder hinge-moment parameters.

An analysis of the data showed that there were no significant changes in the basic rudder-fixed stability derivatives with indicated airspeed, and the rudder-fixed damping characteristics were predictable with sufficient accuracy if the product-of-inertia terms were considered.

Freeing the rudder brought about a reduction in the damping of the oscillation at low speed; this is attributed to the low negative rate of change of rudder hinge-moment coefficient with rudder deflection and a negative rate of change of rudder hinge-moment coefficient with angle of sideslip. The further deterioration with indicated airspeed is attributable to the effects of Mach number and rudder-tab deflection on these hinge-moment parameters.

## INTRODUCTION

A number of military airplanes recently have exhibited poor lateral-oscillation characteristics in high-speed flight. The objectionable motions of the airplane in yaw usually are characterized by constant-amplitude oscillations or by poor damping in conjunction with a short

period. In an effort to determine the causes of these phenomena, the Ames Aeronautical Laboratory has conducted a detailed investigation of the dynamic-lateral-stability characteristics of an airplane for which undesirable oscillatory behavior at high airspeeds had been reported.

At the outset of an investigation of this type there always exists the question of whether the undesirable damping characteristics are due to the basic static-stability derivatives (control fixed), the hinge-moment parameters of the control surfaces (control free), or random separation phenomena. It was therefore convenient to isolate the effect of rudder freedom by determining first the dynamic-stability characteristics with the rudder fixed. These characteristics, which depend primarily on the combination of basic airplane mass and stability parameters, were then analyzed by means of the classic rudder-fixed linearized equations of motion. The effects of rudder freedom, which are functions primarily of the rudder mass and hinge-moment derivatives, were determined and then considered as additional factors or increments which modify the basic rudder-fixed characteristics.

### SYMBOLS

The symbols used in this report are defined in Appendix A.

### DESCRIPTION OF AIRPLANE AND INSTRUMENTATION

The airplane on which the tests discussed herein were conducted was a conventional single-engine low-wing attack airplane. A photograph of the airplane as instrumented for flight and a three-view drawing are presented in figures 1 and 2, respectively. Pertinent physical characteristics of the airplane are given in Appendix B.

During the tests the following rudder configurations were tested:

1. Original rudder (fig. 3)
2. Original rudder with trailing-edge bulb on trim tab, as used on production version of the airplane (fig. 4)
3. Original rudder with upper horn balance area removed (fig. 5)
4. Original rudder with deboost tab (trim tab connected to give deboost motion of  $1^\circ$  of tab for  $2^\circ$  of rudder)

Standard NACA recording instruments were used to record the various quantities presented in this report. The rudder-position recorder, which was connected directly to the bottom of the rudder structure, permitted evaluation of the rudder angle to within  $\pm 0.1^\circ$ .

## TESTS

## Flight Tests

Flight tests were made to determine the lateral-oscillation characteristics of the airplane with the various rudder modifications. The tests were performed in the 4,000- to 10,000-foot pressure-altitude range in the power-on clean configuration at various indicated airspeeds from approximately 200 to 450 miles per hour. The tests were conducted with a normal center-of-gravity location and with no external stores attached.

Prior to each test run the airplane was trimmed (zero control force) in steady, straight, wings-level flight. The oscillations were initiated by release of the rudder control while the airplane was at a moderate angle of sideslip. The instruments were started just before the release of the rudder and left on until the oscillations had completely damped or until approximately 6 cycles were completed. For the rudder-free test the pilot kept his feet clear of the rudder controls until completion of the run. For the rudder-locked test the pilot released a locking pin while in the steady sideslip, and the pin engaged and locked the rudder when it passed through trim position. This locking mechanism is shown in figure 6. The ailerons, whether returned to their trim position and held or left free to oscillate, had no noticeable effect on the yawing motion of the airplane.

## Tests in the Ames 40- by 80-Foot Wind Tunnel

Power-off tests were made in the Ames 40- by 80-foot wind tunnel on the test airplane to determine the rate of change of rudder hinge-moment coefficient with rudder deflection  $C_{h\delta}$  under conditions similar to those of the lateral-oscillation flights. The tests were made at an indicated airspeed of 203 miles per hour at zero yaw with various rudder trim-tab settings. Both the original rudder and the original rudder with the trailing-edge bulb were investigated. These data are presented in figure 7. Values of the rate of change of hinge-moment coefficient with angle of sideslip  $C_{h\beta}$  of -0.038 per radian for the original rudder and -0.045 per radian for the trailing-edge bulb modification were then derived from steady-sideslip data obtained in flight.

## Tests in the Ames 7- by 10-Foot Wind Tunnel

A 0.17-scale powered model of the airplane was tested in the Ames 7- by 10-foot wind tunnel No. 2 to determine the aerodynamic characteristics of the model. The rudder used in these tests was similar to the original rudder configuration of the test airplane, except the rudder tabs were not adjustable.

The basic aerodynamic parameters obtained from these tests and those estimated by the methods of reference 1 are as follows:

|                         |        |                         |        |
|-------------------------|--------|-------------------------|--------|
| $C_{l_p}$ . . . . .     | -0.43  | $C_{n_p}$ . . . . .     | -0.007 |
| $C_{l_r}$ . . . . .     | 0.03   | $C_{n_\beta}$ . . . . . | 0.132  |
| $C_{l_\beta}$ . . . . . | -0.08  | $C_{n_r}$ . . . . .     | -0.160 |
| $C_{Y_\beta}$ . . . . . | -0.802 |                         |        |

## RESULTS AND DISCUSSION

Figure 8 presents typical time histories of the rudder motion and yawing velocities after a disturbance in yaw for various rudder-control procedures. This figure shows that a rudder-free condition existed even though the pilot attempted to hold the pedals fixed. The movement of the rudder with pedals fixed, though small in magnitude (approximately  $\pm 0.3^\circ$  maximum), had an important influence on the damping characteristics of the airplane. The results obtained with this procedure were inconsistent, and are not readily susceptible to analysis by dynamic-stability theory. For the first phase of the investigation, this procedure therefore was discarded in favor of the one in which the rudder was definitely fixed by means of the rudder lock. The characteristics with the rudder-free procedure are considered as the second phase of the investigation.

### Rudder-Fixed Characteristics

The results of the rudder-fixed oscillation flight tests are summarized in figure 9, in which period  $P$ , time to damp to one-half amplitude  $T_{\frac{1}{2}}$ , and cycles to damp to one-half amplitude  $C_{\frac{1}{2}}$  are plotted as a function of indicated airspeed. Plotted on this same figure for the purpose of comparison are the computed values<sup>1</sup> of the three parameters at

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<sup>1</sup>In most theoretical investigations, the effect of the product-of-inertia terms resulting from the inclination of the principal longitudinal axis of the airplane has been considered small. However, Sternfield recently has shown in reference 2 that neglect of these terms can lead to considerable error in the prediction of the oscillatory divergency boundaries for modern airplanes. Computations for the test airplane based on the equations of reference 3, which include the effect of the product-of-inertia terms, give the value of 0.75 for  $C_{\frac{1}{2}}$  shown in figure 9. A value of 0.67 was calculated by the method of reference 4 which neglects these terms. Inclusion of the product-of-inertia terms had negligible effect on the computed period for the test airplane.

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an airspeed of 210 miles per hour and an extrapolation of these characteristics to higher speeds based on the assumption of a constant value of the stability derivatives throughout the speed range.

In view of the deterioration in damping at higher speeds, observed with rudder free (to be discussed later), it was considered possible that there might be a significant Mach number effect on the stability derivatives. The fact that this is not so over the Mach number range of the flight tests with regard to any of the stability derivatives which affect the rudder-locked oscillation is shown by the comparison of the variation of the experimental and theoretical characteristics with airspeed in figure 9. The inverse variation of  $P$  and  $T_{\frac{1}{2}}$  with speed and the constant value of  $C_{\frac{1}{2}}$  shown in this figure is in accordance with the control-locked stability equations for a constant value of the stability derivatives. The correspondence between the experimental and predicted oscillation characteristics over the range of the tests is considered to be within the experimental accuracy, an indication that the assumption of a constant value of the stability derivatives over the Mach number range was a sound one and that there are no unaccounted factors affecting the oscillation characteristics with rudder fixed. From this it is concluded that the poor damping characteristics of the test airplane arise from the rudder hinge-moment characteristics.

#### Rudder-Free Characteristics

During the rudder-free flight tests, three types of yawing oscillations were encountered. These oscillations have been termed completely damped, incompletely damped, and increasing amplitude. Typical yawing-velocity records of these types are presented in figure 10. The damping characteristics derived from records of this type for the various rudder modifications are summarized in figure 11.

In order to compare the experimental results with those predicted from the values of the rudder hinge-moment parameters produced by the various rudder modifications, the variation of the damping characteristics of the test airplane for a range of values of  $C_{n\beta}$  and  $C_{n\delta}$  has been computed by the basic method of reference 4.<sup>2</sup> Figure 12 shows the boundaries for the various types of lateral oscillation and figure 13 shows the variation of the period and cycles to damp to one-half amplitude computed by this method.

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<sup>2</sup>The computations were made by the simplified method of reference 5, as outlined in Appendix C, which makes practical the consideration of the effects of product-of-inertia terms.

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The values of  $C_{h\delta}$  and  $C_{h\beta}$  at a speed of 203 miles per hour were available from the 40- by 80-foot wind-tunnel tests and the flight tests for the original rudder and for the original rudder with a trailing-edge bulb. Application of these data at the tab setting used in flight for the wings level, trimmed condition ( $2.2^\circ$  left for the original rudder and  $0.8^\circ$  left for the original rudder with trailing-edge bulb) results in the following comparison between the predicted and measured oscillation characteristics.

| Rudder configuration            | Flight Tests |                              | Predicted from full-scale $C_h$ data |                              |
|---------------------------------|--------------|------------------------------|--------------------------------------|------------------------------|
|                                 | Period (sec) | Cycles to one-half amplitude | Period (sec)                         | Cycles to one-half amplitude |
| Original rudder                 | 2.13         | 1.75                         | 1.95                                 | 1.35                         |
| Trailing-edge bulb modification | 2.30         | 1.15                         | 2.03                                 | 1.01                         |

In the case of the original rudder with trailing-edge bulb, the comparison between the computed and measured damping is good. The likely reason for the less satisfactory agreement in the case of the original rudder is evident from figure 13, which shows that with the small value of  $C_{h\delta}$  existent on the original rudder, a small inaccuracy in the evaluation of  $C_{h\delta}$  or  $C_{h\beta}$  would cause appreciable changes in the cycles to damp to one-half amplitude. Another source of error not evident from this figure is inaccuracy in the evaluation of the rudder damping parameters<sup>3</sup>  $C_{hD\delta}$ .

This term, which is relatively unimportant when  $C_{h\delta}$  is high, increases in importance at the lower values of  $C_{h\delta}$  and can significantly affect the location of the knee of the curves of figure 13(a). It can be concluded from the foregoing that the lateral-oscillation characteristics can be satisfactorily predicted by available methods, provided the combination of hinge-moment parameters is not such as to put the airplane in the knee or beyond the knee of the curves of figure 13(a).

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<sup>3</sup>The value of  $C_{hD\delta}$  of -0.154 used in this investigation was obtained from reference 6.

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The important influence of tab setting is evident from figure 7. The change in  $C_{h\delta}$  between  $\pm 4^\circ$  tab setting is sufficient to increase the cycles to damp to one-half amplitude as much as 50 percent on the original rudder and 10 percent on the rudder with the trailing-edge bulb as shown in figure 13(a). This effect was exemplified in flight, as indicated by the curves in figure 11. Above 400 miles per hour, the damping characteristics are considerably inferior for the out-of-trim tab setting, in comparison with the characteristics for the in-trim tab setting. It is evident that a conservative design must allow a reasonable margin for these changes in  $C_{h\delta}$  due to tab setting, if undesirable damping characteristics are to be avoided.

No quantitative data were available on  $C_{h\delta}$  and  $C_{h\beta}$  with the deboost tab or the horn-removed modifications. However, it is known that the former modification will increase  $C_{h\delta}$  negatively and the latter will increase  $C_{h\beta}$  positively and  $C_{h\delta}$  negatively. Both these changes are such as to move the rudder characteristics into the better damping range. Figure 11 shows that the flight results are in conformity with this trend. The deboost tab modification is of special interest, since it furnishes an expedient by which a rudder which has an undesirable  $C_{h\delta}$  and  $C_{h\beta}$  combination, insofar as damping is concerned, can readily be adjusted for more favorable characteristics.

On all of the rudder modifications tested, except with the upper horn modification, there is observed a gradual deterioration of the damping of the airplane with speed. As previously mentioned, the rudder-fixed tests show that this deterioration is not traceable to any control-fixed stability parameters. It can be inferred, therefore, that the effect is due to a change in  $C_{h\delta}$  and  $C_{h\beta}$  with Mach number. No experimental data are available to verify this inference on the rudder used on the test airplane, but this trend has been observed in high Mach number tests of other control surfaces. In particular, if the trailing-edge angle is large, it can be anticipated that  $C_{h\delta}$  will increase positively and  $C_{h\beta}$  will increase negatively with Mach number, a trend which figure 13 shows will move the rudder into the decreased damping range. Sound design practice, therefore, calls for an allowance for changes of this type in selecting the low-speed parameters by methods of references 4 and 5.

#### CONCLUDING REMARKS

The current trend of airplane design which leads to intentional selection of a low rate of change of rudder hinge-moment coefficient with rudder deflection  $C_{h\delta}$  and a negative rate of change of rudder hinge-moment



coefficient with angle of sideslip  $C_{h\beta}$  is such as to invite increasing amplitude or poorly damped oscillations. It is therefore important that the damping characteristics be checked by the methods of Greenberg and Sternfield or, if product-of-inertia terms are found to be important, by the methods of Neumark. Allowances should be made for the effect of tab deflection and Mach number on the hinge-moment parameters.

Ames Aeronautical Laboratory,  
National Advisory Committee for Aeronautics,  
Moffett Field, Calif., Dec. 19, 1947.

## APPENDIX A

## SYMBOLS

|                      |  |
|----------------------|--|
| $S$                  | wing area, square feet   |
| $V$                  | free-stream airspeed, feet per second  |
| $V_i$                | indicated airspeed, miles per hour   |
| $b$                  | wing span, feet  |
| $b_r$                | span of rudder, feet   |
| $m$                  | mass of the airplane, slugs  |
| $m_r$                | mass of the rudder, slugs  |
| $k_z$                | radius of gyration of airplane about vertical (Z) axis, feet   |
| $k_r$                | radius of gyration of rudder about hinge axis, feet  |
| $P$                  | period of oscillations, seconds  |
| $T_{\frac{1}{2}}$    | time required for motions to decrease to one-half amplitude, seconds   |
| $C_{\frac{1}{2}}$    | cycles required for motions to decrease to one-half amplitude  |
| $t$                  | time, seconds  |
| $A, B, C,$<br>$E, F$ | coefficients of the rudder-free lateral-stability equation   |
| $f, h$               | coefficient of the rudder-fixed quadratic factor $\lambda^2 + f\lambda + h$ for which the roots correspond to the apparent lateral oscillation |
| $\lambda$            | root of the stability determinant  |
| $ib'$                | imaginary portion of complex root of the stability determinant   |
| $a'$                 | real portion of the complex root of the stability determinant  |

|               |  |
|---------------|--|
| $q$           | dynamic pressure, pounds per square foot $\left(\frac{1}{2}\rho V^2\right)$  |
| $\rho$        | mass density of air, slugs per cubic foot  |
| $\mu$         | airplane relative-density factor $(m/\rho S b)$  |
| $\mu_r$       | rudder relative-density factor $(m_r/\rho b_r \bar{c}_r^2)$  |
| $\bar{c}_r$   | mean aerodynamic chord of rudder, feet   |
| $\beta$       | angle of sideslip, radians   |
| $\delta$      | rudder deflection, radians   |
| $r$           | yawing angular velocity, radians per second  |
| $C_Y$         | lateral-force coefficient (lateral force/ $qS$ )   |
| $C_n$         | yawing-moment coefficient (yawing moment/ $qSb$ )  |
| $C_l$         | rolling-moment coefficient (rolling moment/ $qSb$ )  |
| $C_h$         | hinge-moment coefficient (hinge moment/ $q b_r \bar{c}_r^2$ )  |
| $C_{n\beta}$  | rate of change of yawing-moment coefficient with angle of sideslip $(\partial C_n / \partial \beta)$                             |
| $C_{Y\beta}$  | rate of change of lateral-force coefficient with angle of sideslip $(\partial C_Y / \partial \beta)$                             |
| $C_{l_r}$     | rate of change of rolling-moment coefficient with yawing angular-velocity factor $\left[\partial C_l / \partial (rb/2V)\right]$  |
| $C_{l_p}$     | rate of change of rolling-moment coefficient with rolling angular-velocity factor $\left[\partial C_l / \partial (pb/2V)\right]$ |
| $C_{l\beta}$  | rate of change of rolling-moment coefficient with angle of sideslip $(\partial C_l / \partial \beta)$                            |
| $C_{n_r}$     | rate of change of yawing-moment coefficient with yawing angular-velocity factor $\left[\partial C_n / \partial (rb/2V)\right]$   |
| $C_{n\delta}$ | rate of change of yawing-moment coefficient with rudder deflection $(\partial C_n / \partial \delta)$                            |
| $C_{h\beta}$  | rate of change of rudder hinge-moment coefficient with angle of sideslip $(\partial C_h / \partial \beta)$                       |

- $C_{h_r}$  rate of change of rudder hinge-moment coefficient with yawing angular-velocity factor  $\left[ \frac{\partial C_h}{\partial (rb/2V)} \right]$
- $C_{h_\delta}$  rate of change of rudder hinge-moment coefficient with rudder deflection ( $\partial C_h / \partial \delta$ )
- $C_{h_{D\delta}}$  rate of change of rudder hinge-moment coefficient with rudder angular-velocity factor  $\left\{ \frac{\partial C_h}{\partial \left[ \frac{(d\delta/dt)b}{2V} \right]} \right\}$

## APPENDIX B

## PHYSICAL CHARACTERISTICS OF THE TEST AIRPLANE

## General

|   |                             |
|---|-----------------------------|
| Test gross weight . . . . .   | 13,000 lb                   |
| Moment of inertia in yaw  |                             |
| about principal axis . . . . .  | 36,340 slug-ft <sup>2</sup> |
| Moment of inertia in roll   |                             |
| about principal axis . . . . .  | 13,980 slug-ft <sup>2</sup> |
| Angle between reference axis and principal axis, positive when reference axis is above principal axis at nose . . . . . | 2.4°                        |
| Distance from airplane center of gravity (0.25 M.A.C.) to rudder hinge line . . . . .                                   | 23.0 ft                     |

## Wing

|   |                         |
|---|-------------------------|
| Area . . . . .                              | 400 sq ft               |
| Span . . . . .                              | 50.0 ft                 |
| Aspect ratio . . . . .                      | 6.25                    |
| Taper ratio . . . . .                       | 0.503                   |
| Mean aerodynamic chord . . . . .            | 8.34 ft                 |
| Dihedral . . . . .                          | 6.0°                    |
| Incidence (root-chord station 30) . . . . . | 3.97°                   |
| Geometric twist . . . . .                   | 4.22°                   |
| Root section . . . . .                      | NACA 2417 (station 30)  |
| Tip section . . . . .                       | NACA 4413 (station 300) |

## Vertical tail

|                |             |
|----------------|-------------|
| Area . . . . . | 35.01 sq ft |
| Span . . . . . | 7.67 ft     |

|                        |  |
|------------------------|--|
| Aspect ratio           | 1.68   |
| Taper ratio            | 0.549  |
| Mean aerodynamic chord | 4.78 ft  |
| Offset                 | 3.0° from station 45 to tip                      |
| Root section           | NACA 0013.44-64 modified to 12-percent thickness |
| Tip section            | NACA 0012-64 modified to 10.71-percent thickness |

#### Rudder

|                                  |                                |
|----------------------------------|--------------------------------|
| Area aft of hinge line           | 18.99 sq ft                    |
| Span                             | 9.67 ft                        |
| Mean aerodynamic chord           | 2.08 ft                        |
| Chord aft of hinge line, percent |                                |
| fixed surface chord              | 40                             |
| Total balance area               |                                |
| Original rudder                  | 4.82 sq ft                     |
| Original rudder with the upper   |                                |
| horn removed                     | 3.79 sq ft                     |
| Nose gap                         | fabric sealed                  |
| Contour aft of hinge line        | straight                       |
| Travel                           | (approximately) $\pm 25^\circ$ |
| Included trailing-edge angle     | 13°                            |

#### Tabs

##### Trim

|                                 |                                |
|---------------------------------|--------------------------------|
| Area aft of hinge line          | 1.91 sq ft                     |
| Span                            | 4.08 ft                        |
| Chord, percent of control chord | 6.99                           |
| Travel                          | (approximately) $\pm 10^\circ$ |
| Balance ratio (tab throw        |                                |
| per surface throw)              |                                |
| Original rudder                 | 0                              |
| Deboost modification            | 0.5                            |

##### Spring

|                                 |                                |
|---------------------------------|--------------------------------|
| Area aft of hinge line          | 1.00                           |
| Span                            | 2.59 ft                        |
| Chord, percent of control chord | 7.01                           |
| Travel                          | (approximately) $\pm 15^\circ$ |
| Spring constant, rudder         |                                |
| hinge moment per tab angle      | 5.78 ft-lb per degree          |

##### Moment of inertia of the rudder

|                                |                           |
|--------------------------------|---------------------------|
| system about hinge line        | 2.27 slug-ft <sup>2</sup> |
| Rudder frictional hinge moment | 1.2 ft-lb                 |

## Engine

|                                |                     |
|--------------------------------|---------------------|
| Type . . . . .                 | Wright R-3350-8     |
| Propeller gear ratio . . . . . | 0.4375              |
| Normal horse power ratings     |                     |
| Sea level . . . . .            | 2100 hp at 2400 rpm |
| 3800 feet . . . . .            | 2100 hp at 2400 rpm |

## Propeller

|                            |                                 |
|----------------------------|---------------------------------|
| Type . . . . .             | Curtiss Electric                |
| Number of blades . . . . . | four                            |
| Diameter . . . . .         | 13 ft 0 in.                     |
| Blade design . . . . .     | X836-14C2-18R1<br>cuffs removed |

## APPENDIX C

## CALCULATION METHODS

The method employed in the present investigation for predicting the rudder-free dynamic lateral stability of the airplane was taken from reference 5. It makes use of the coefficients of the rudder-locked quadratic factor  $\lambda^2 + f\lambda + h$ , for which the roots correspond to the motion which is readily apparent to the pilot. The substance of this procedure is the neglecting of the effect of  $C_{Y\beta}$  on rudder-motion terms during the determinant expansion process while retaining  $C_Y$  effects on other terms. Application of this procedure, using rudder-fixed coefficients and not considering rolling, yields an equation of the form:

$$A\lambda^4 + B\lambda^3 + C\lambda^2 + E\lambda + F = 0 \quad (1)$$

where

$$A = 1$$

$$B = f - \frac{C_{hD\delta}}{4\mu_r} \left( \frac{b}{k_r} \right)^2$$

$$C = h + \frac{C_{n\delta}}{2\mu} \left( \frac{b}{k_z} \right)^2 - \frac{C_{h\delta}}{2\mu_r} \left( \frac{b}{k_r} \right)^2 - \frac{f(C_{hD\delta})}{4\mu_r} \left( \frac{b}{k_r} \right)^2 + \frac{l_{x_r} C_{n\delta}}{2\mu(k_z)^2} \left( \frac{b}{k_r} \right)^2$$

$$E = \frac{f(C_{h\delta})}{2\mu_r} \left(\frac{b}{k_r}\right)^2 - \frac{h(C_{hD\delta})}{4\mu_r} \left(\frac{b}{k_r}\right)^2 - \frac{C_{h_r} C_{n\delta}}{8\mu_r} \left(\frac{b}{k_z}\right)^2 \left(\frac{b}{k_r}\right)^2$$

$$F = \frac{C_{h\beta} C_{n\delta}}{4\mu_r \mu} \left(\frac{b}{k_r}\right)^2 \left(\frac{b}{k_z}\right)^2 - \frac{h(C_{h\delta})}{2\mu_r} \left(\frac{b}{k_r}\right)^2$$

The roots of equation (1) which define the apparent lateral oscillations are of the form  $\lambda = a' + ib'$ . These roots are used in the following equations to determine the period and time to damp to one-half amplitude:

$$P = \frac{2\pi}{b'} \frac{b}{V} \quad (2)$$

$$T_{\frac{1}{2}} = \frac{-\log_e 0.5}{a'} \quad (3)$$

The relationship between  $C_{h\delta}$  and  $C_{h\beta}$  which defines the boundary for divergence is obtained by setting  $F = 0$  and that for oscillations of increasing amplitude is found by setting Routh's discriminant

$$R = BCE - AE^2 - FB^2 = 0$$

The incomplete damping boundary, which defines a region between damped and increasing-amplitude oscillations in which constant-amplitude oscillations occur, is the envelope of the boundaries for increasing oscillations for various values of  $C_{hD\delta}$ .

The coefficients  $f$  and  $h$  of the rudder-locked quadratic factor, which are used in the present investigation, were obtained from the rudder-locked flight-test data presented in figure 9. By the use of the quadratic formula, equations (2) and (3) can be transformed for the rudder-locked case to the form

$$f = \frac{2 \log_e 0.5}{T_{\frac{1}{2}}} \frac{b}{V} \quad (4)$$

and

$$h = \frac{4\pi^2}{P^2} \frac{b^2}{V^2} + \frac{f^2}{4} \quad (5)$$

Values of  $T_{\frac{1}{2}}$ ,  $P$ , and  $V$  from figure 9 can then be substituted in equations (4) and (5) to evaluate  $f$  and  $h$ . Rudder-locked oscillation

flight-test data usually are not available for use in the prediction of rudder-free characteristics. In that case,  $a'$  and  $b'$  (and hence  $f$  and  $h$ ) can be evaluated from wind-tunnel data or other information by a method such as that given in reference 3, in which the effects of rolling and the product of inertia due to the inclination between the wind axis and the principal axis are considered.



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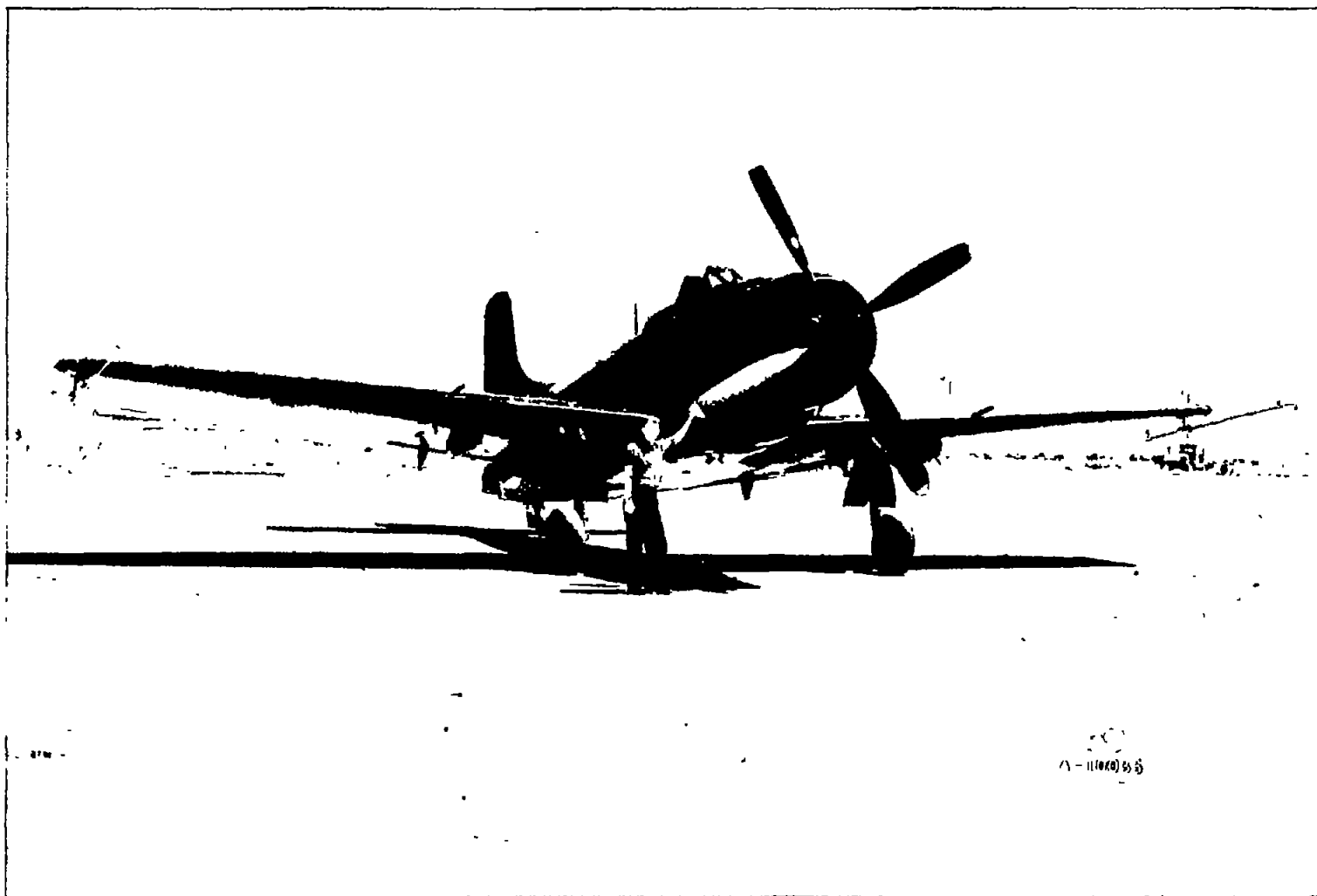
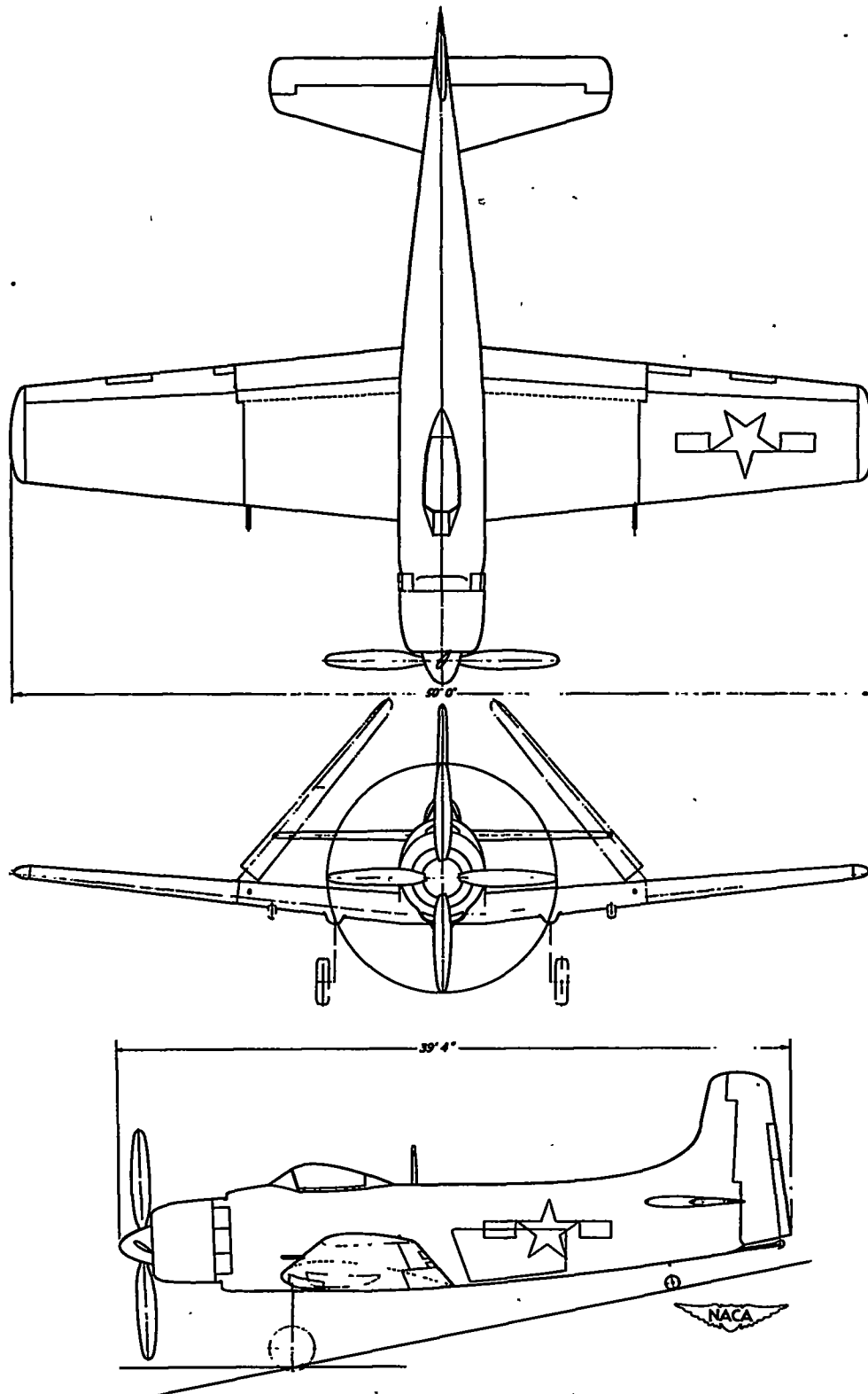


Figure 1.— The test airplane instrumented for flight.





*Figure 2.- Three-view drawing of the test airplane.*

All dimensions in inches

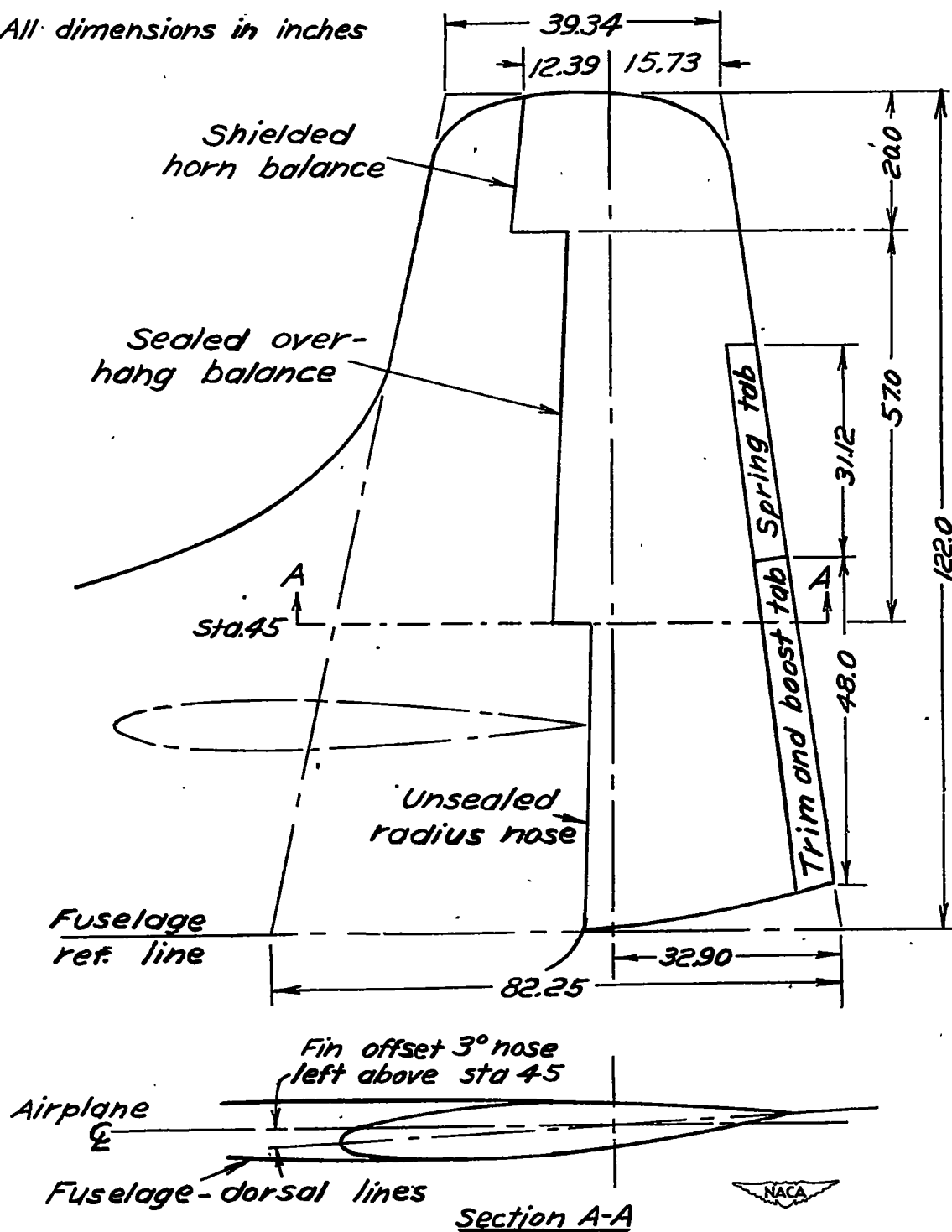
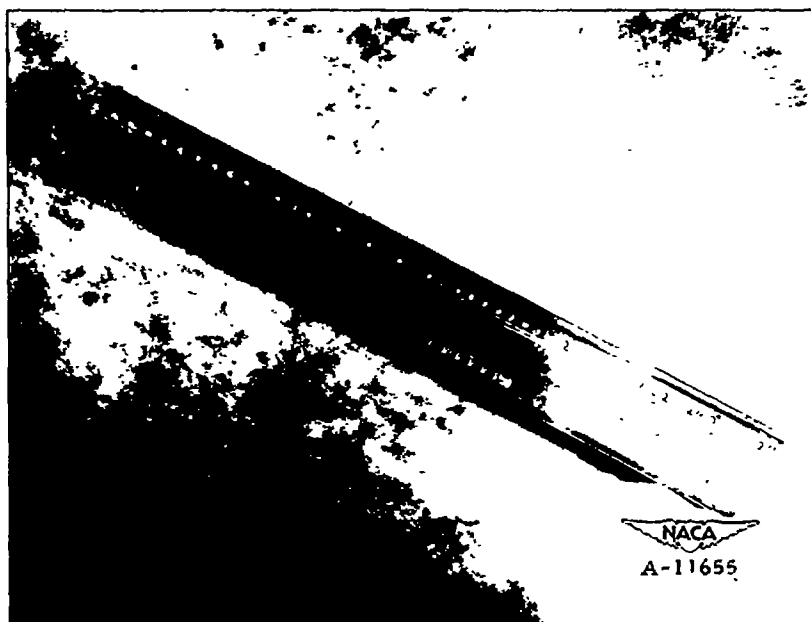
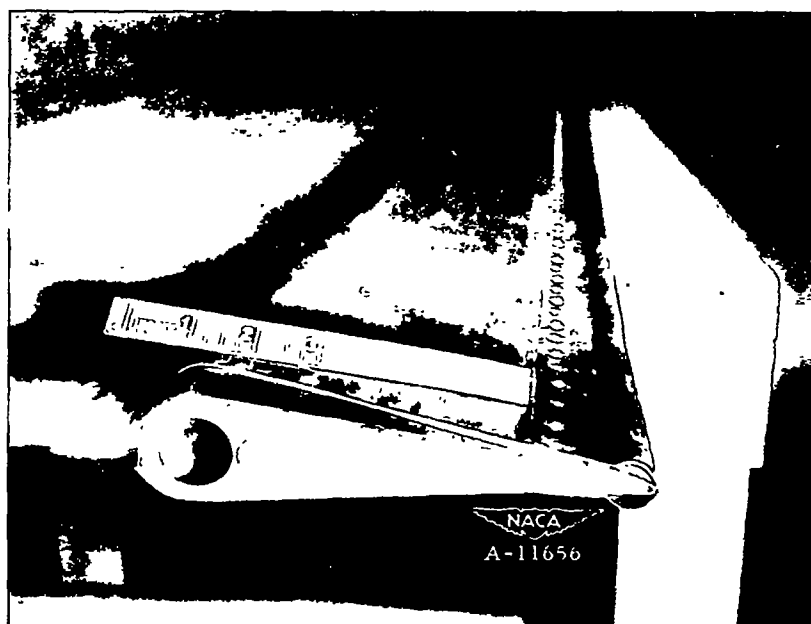


Figure 3.- Vertical tail with the original rudder.



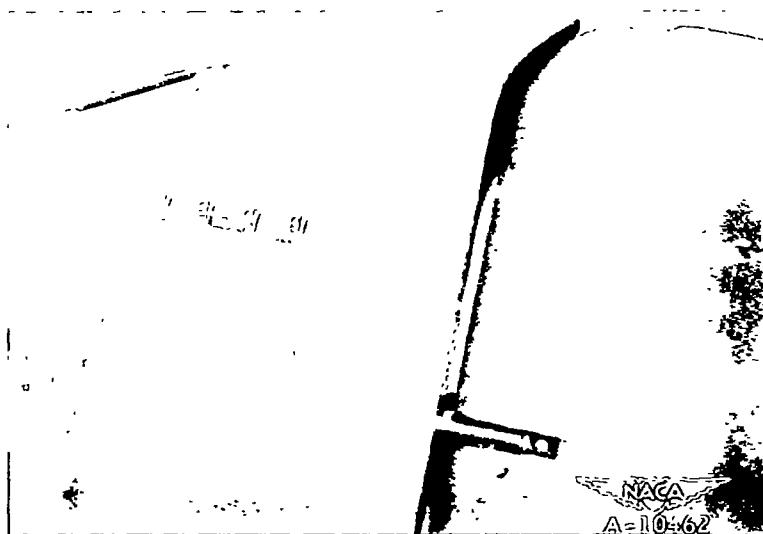
(a) Top view.



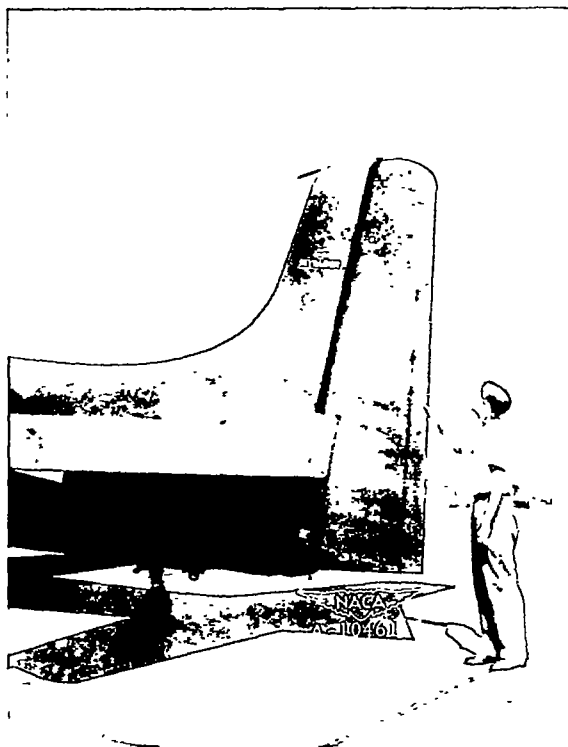
(b) End view.

Figure 4.— Rudder trim tab with the trailing-edge bulb attached.





(a) Close up.



(b) Side view.

Figure 5.— Vertical tail with the horn-balance area removed from the rudder.







(a) Three-quarter rear view.



(b) Rear view.

Figure 6.- Mechanism for locking the rudder.



Rudder angle  $2.5^\circ$  left

Yaw  $0^\circ$

Indicated airspeed 203 mph

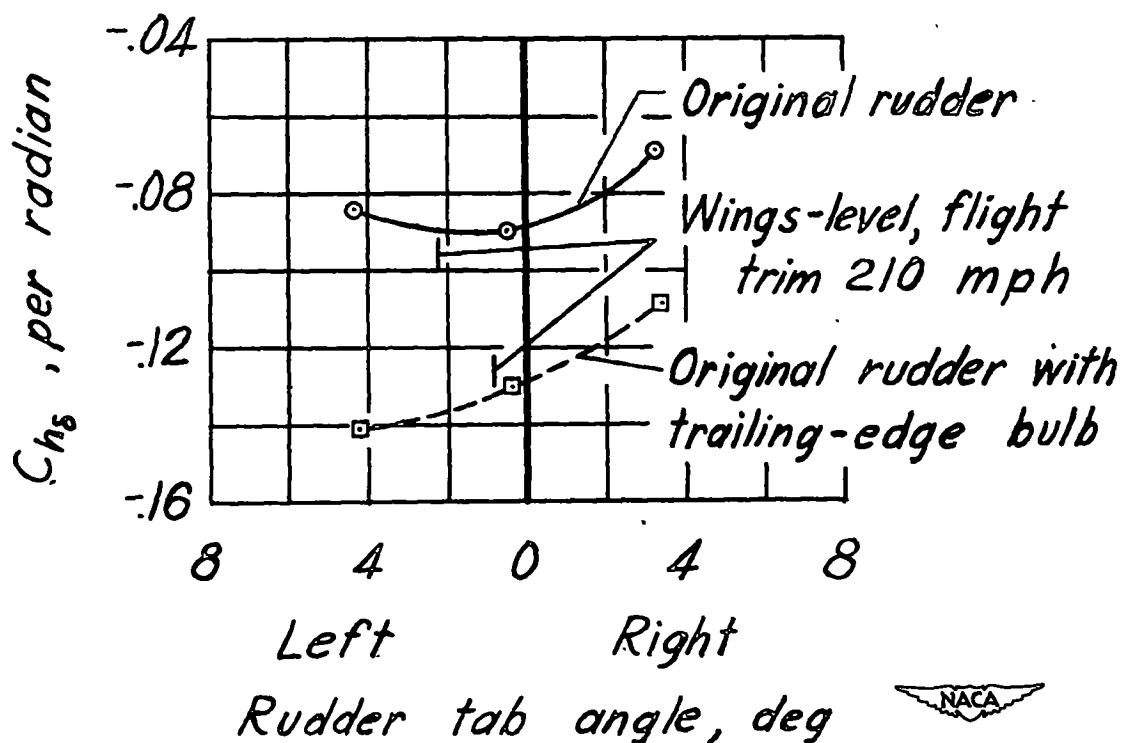


Figure 7.- Variation of  $C_{h8}$  with rudder tab angle as measured in the 40-by 80-foot wind tunnel.

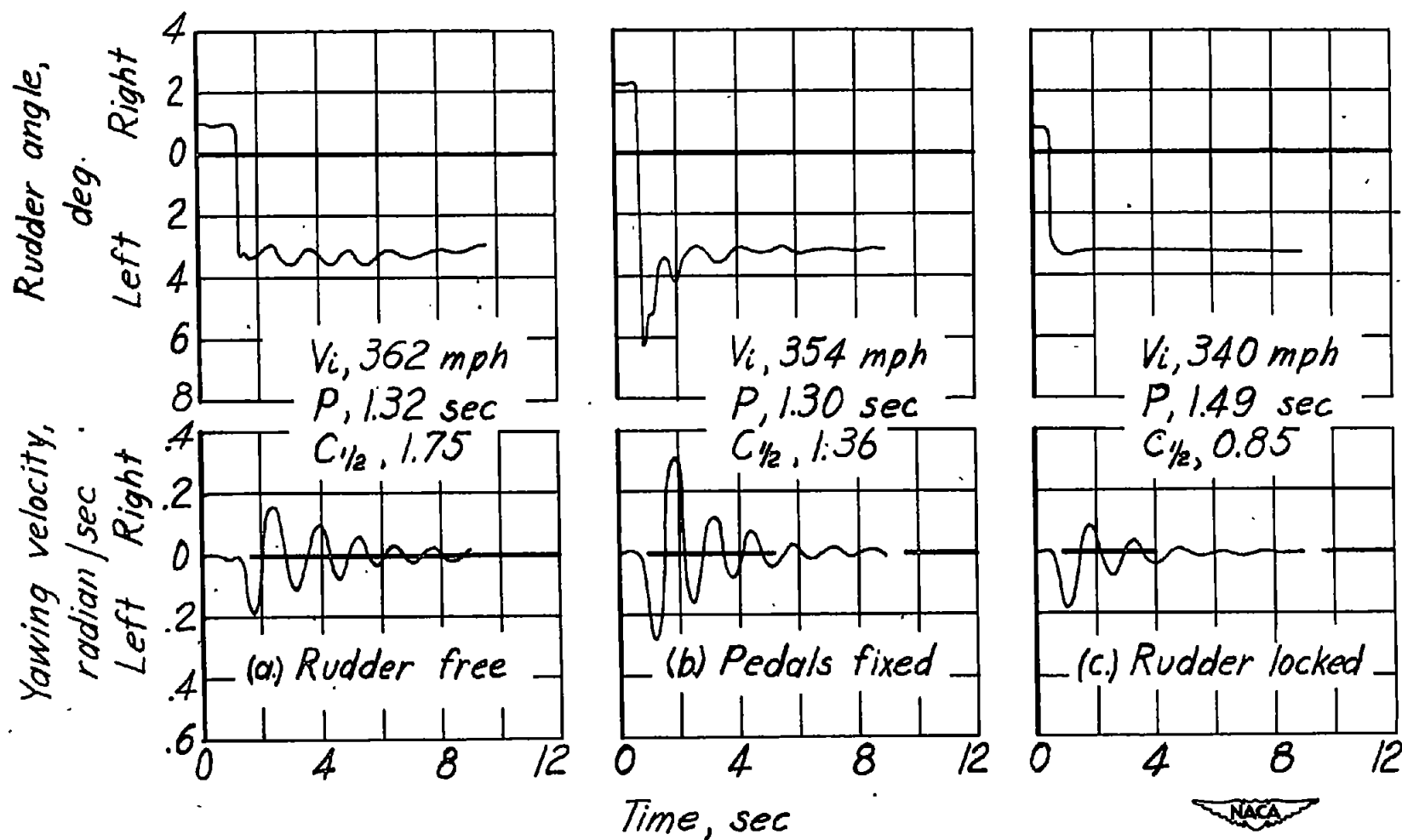


Figure 8.- Time histories showing the effect of rudder-control procedure on lateral-oscillation characteristics.

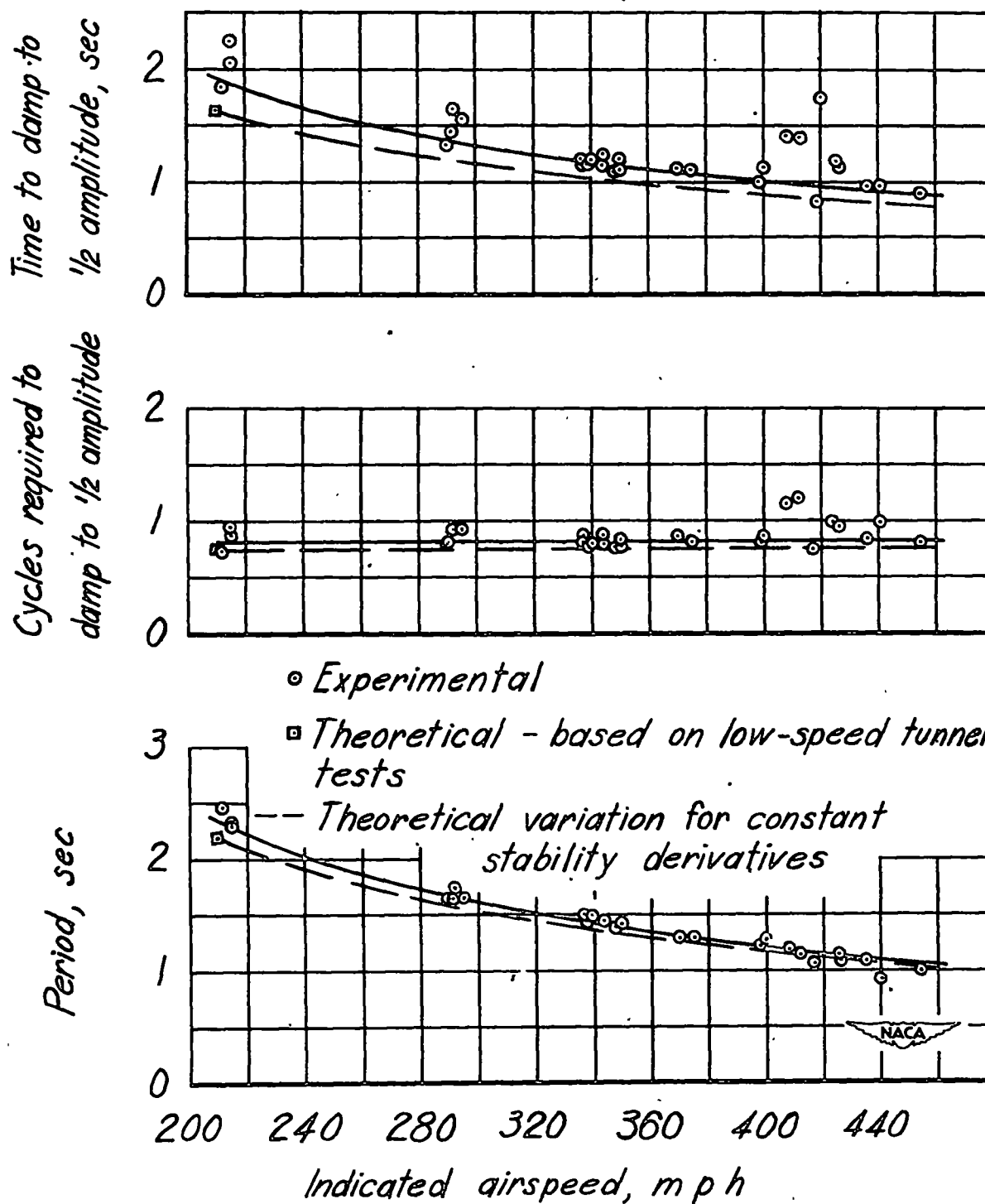


Figure 9.- Variation of period and damping with air-speed, rudder locked.

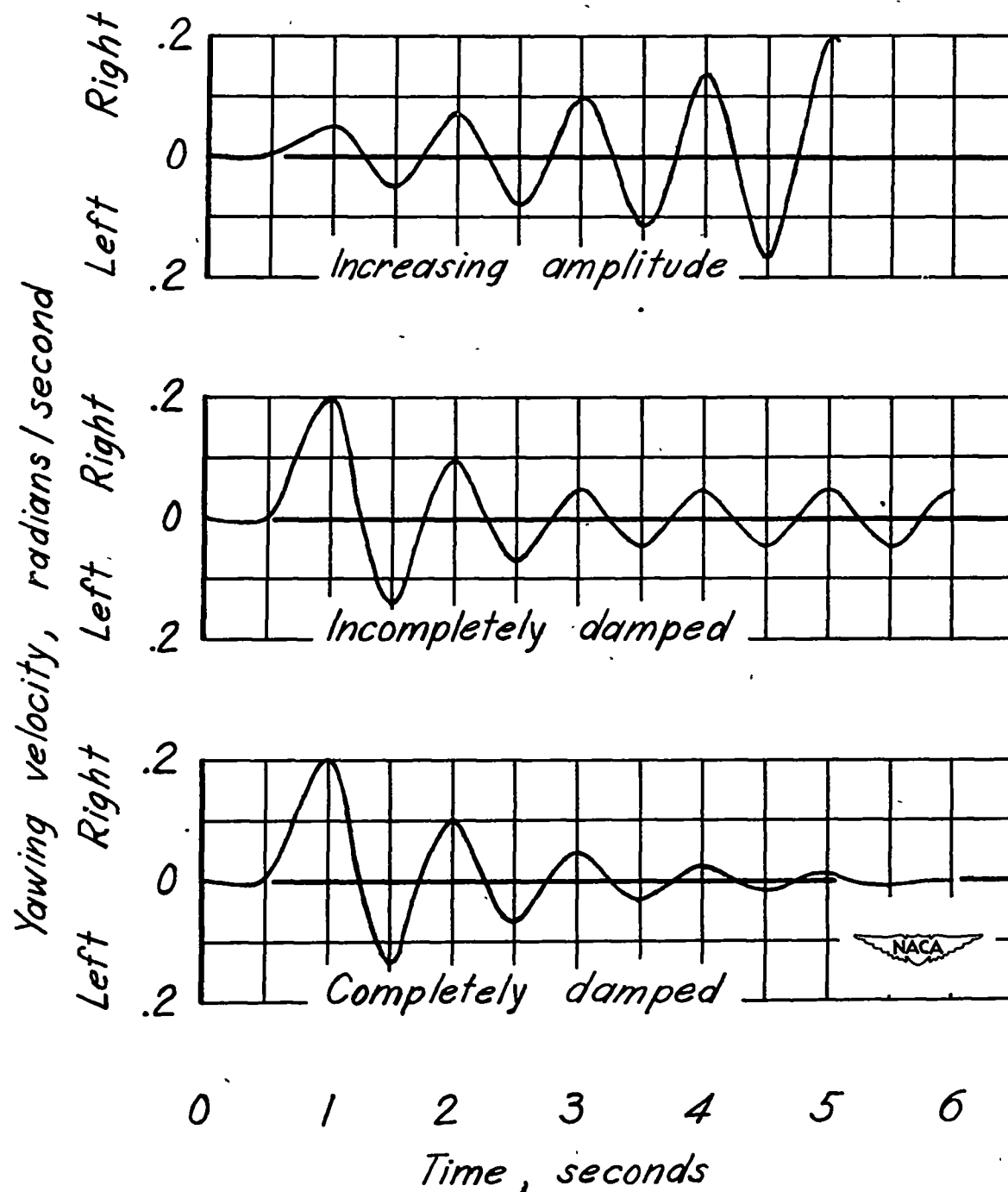


Figure 10- Typical yawing-velocity time histories of the various types of oscillations encountered.

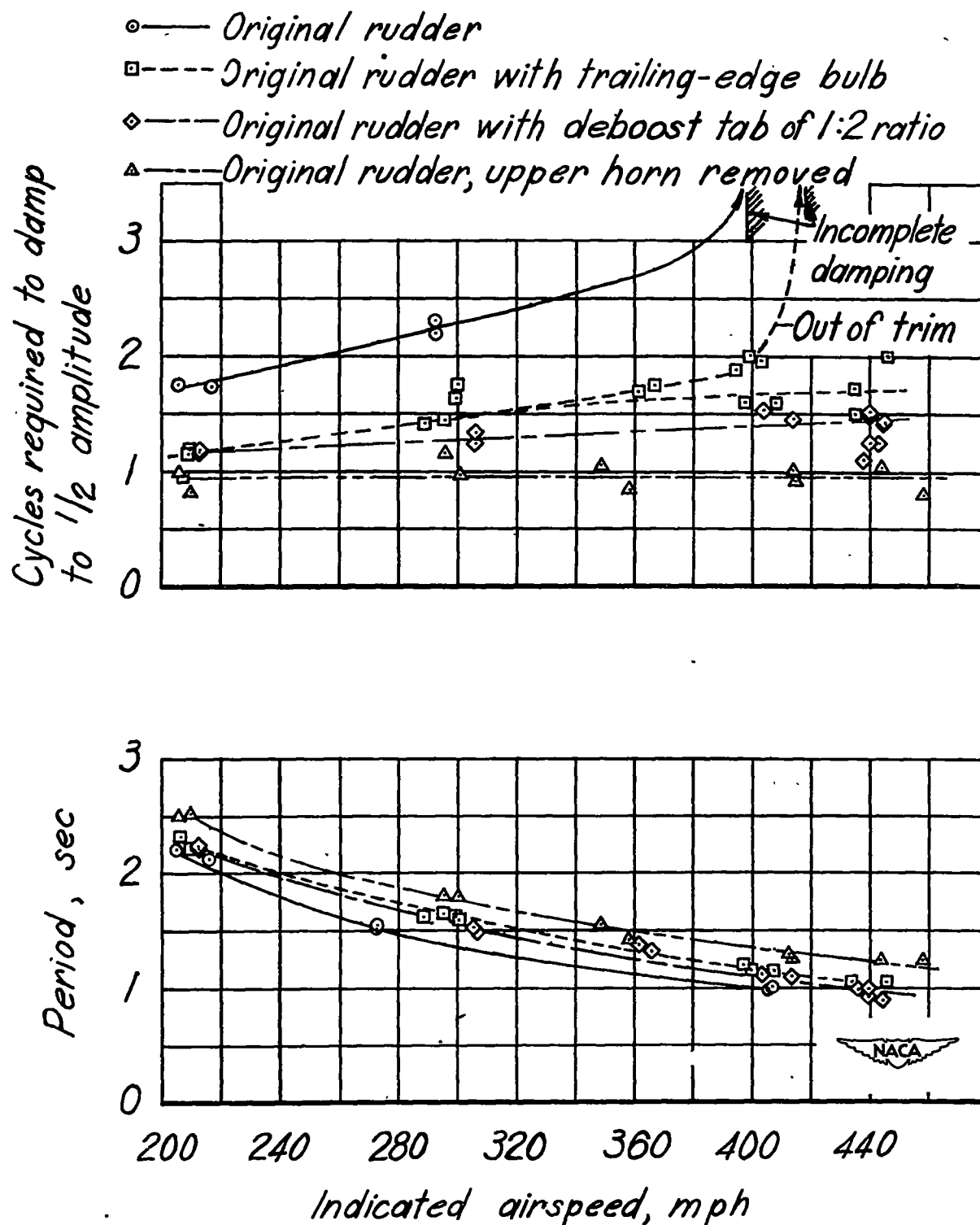


Figure 11.- Variation of period and damping with airspeed, rudder free.



- Original rudder, full-scale data
- Trailing-edge bulb modification, full-scale data

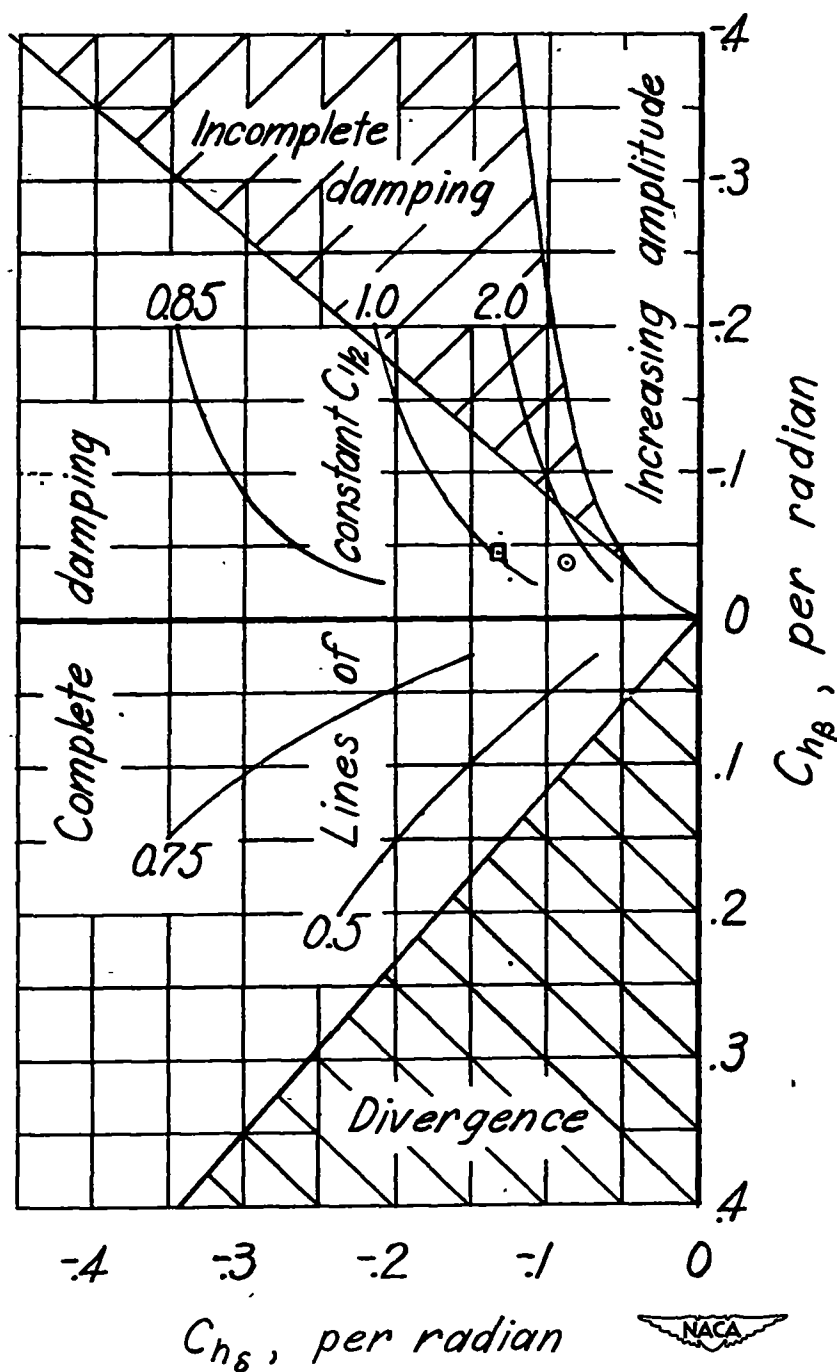


Figure 12.- Boundaries for the various types of rudder-free lateral oscillations.

Trim-tab setting

- +4°
- -4°
- +4°
- -4°

Original rudder

Trailing-edge  
bulb modification

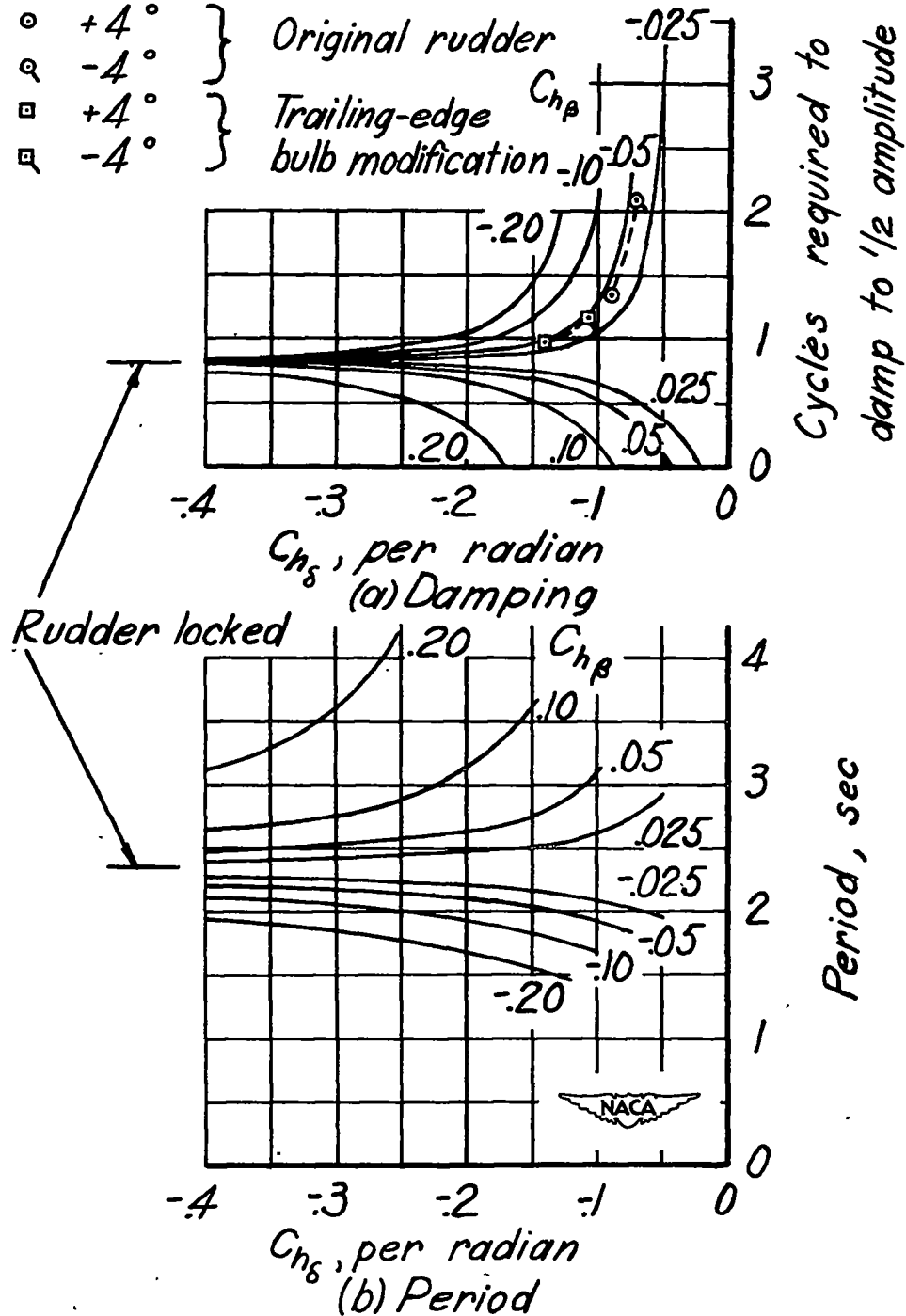


Figure 13.- Variation of period and damping with  $C_{h\delta}$  for various values of  $C_{h\beta}$ ;  $V_i$ , 210 mph.